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Conduction heat and entropy transfer in a semi-infinite medium and wall with a combined periodic heat flux and convective boundary condition

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Abstract

The influence of combined periodic heat flux and convective boundary condition on heat conduction and entropy transfer through semi-infinite and finite media is analytically studied in this work. The Cauchy's residue theorem is utilized to obtain the analytical solutions. It is found that fluctuations in the temperature inside each medium decreases as the Biot number (*Bi*) increases or as the heat transfer parameter and the thermal oscillation parameter decrease. Also, the amplitude of the steady periodic noise in heat and entropy transfer is found to decrease as *Bi* increases or as the heat transfer parameter decreases. Moreover, it is found that the rate of entropy transfer to both media reaches maximum values at critical times lower than the time needed for both the applied heat flux and the rate of heat transferred to reach their maximum values. Finally, it is found that the decrease in the frequency of the applied heat flux and the increases in thermal diffusivity of the medium diminish the noise in temperature and both heat and entropy transfer without affecting their mean or steady state values. As such, this work paves a way on controlling the noise in thermal characteristics of solid media.

Keywords: Conduction; Heat transfer; Oscillation; Semi-infinite medium; Wall; Laplace transform; Entropy transfer

1. Introduction

Heat conduction in semi-infinite or finite media subject to various thermal boundary conditions is extensively studied in literature [1–3]. Among these conditions are those that change with time on a periodic basis which are of great importance to many engineering applications such as HVAC (heating, ventilating and air conditioning) and conservation of energy in buildings applications. For example, the free stream temperature of the outside air is considered to vary with time periodically in many of HVAC researches [4,5]. However, studies about periodic (with respect to time) heat flux boundary conditions are rarely found in the literature despite its importance in many applications like, for example, outer space cooling devices, solar energy applications, material processing using laser beams, microwave heating and in thermal engineering measurements.

Solar radiations, laser or microwaves provide heat fluxes that vary periodically with time. As an example, solar radiations have relatively short frequency. However, laser irradiations or microwaves have very high frequencies. Among the few works that considered periodic thermal disturbance due to laser heating is the work of Yen and Wu [6]. They utilized a periodic thermal disturbance and surface radiation in modeling heat conduction in a finite medium. Recently, Cossali [7] studied the effect of periodic heat flux boundary condition on forced convection over semi-infinite plate. In the few found studies that considered periodic (with respect to time) heat flux boundary conditions [7,8], none of these researches were interested in conduction heat transfer or in the transport of entropy while the other researches are of experimental type. To the author best knowledge, the only study found which deals with entropy transfer during conduction process is for Strub et al. [9]. They considered the second law of the thermodynamic analysis of periodic heat conduction through a wall due to a periodic convective boundary condition. On the other hand, they obtained approximate solution for heat and entropy transfer prior to the application of the second law of thermodynamics.

In this work, exact solution for the temperature field due to a combined periodic heat flux with respect to time and a convec-

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Nomenclature

Bi	Biot number for the slab wall	
Bi^*	Biot number for the semi-infinite medium	
c	specific heat of the medium	$\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$
k	thermal conductivity	$\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$
H	wall thickness	m
h	convection coefficient	$\mathrm{W\,m^{-2}\,K^{-1}}$
P	thermal oscillation parameter	
q	heat flux	$\mathrm{W\,m^{-2}}$
q_d	disturbance in the applied heat flux	$\mathrm{W\,m^{-2}}$
q_m	mean value of the applied heat flux	$\mathrm{W\,m^{-2}}$
q_{mean}	mean value for heat transferred to the medium	$\mathrm{W\,m^{-2}}$
q_s	external applied oscillatory heat flux	$\mathrm{W\,m^{-2}}$
q_o	amplitude of the disturbance in the applied heat flux	$\mathrm{W\,m^{-2}}$
T	dimensional temperature	K
T_∞	ambient temperature	K
t	time	s
Y	dimensionless normal coordinates	
y	dimensional normal coordinates	m

Greek symbols

α	thermal diffusivity	$\mathrm{m^2\,s^{-1}}$
η	dimensionless y-coordinate for the semi-infinite medium	
λ	heat transfer parameter	
θ	excess in temperature	K
θ_m	mean excess in temperature	K
θ_d	disturbance excess temperature	K
ρ	density of the medium	$\mathrm{kg\,m^{-3}}$
τ	dimensionless time	
ω	frequency of the oscillation	$\mathrm{s^{-1}}$

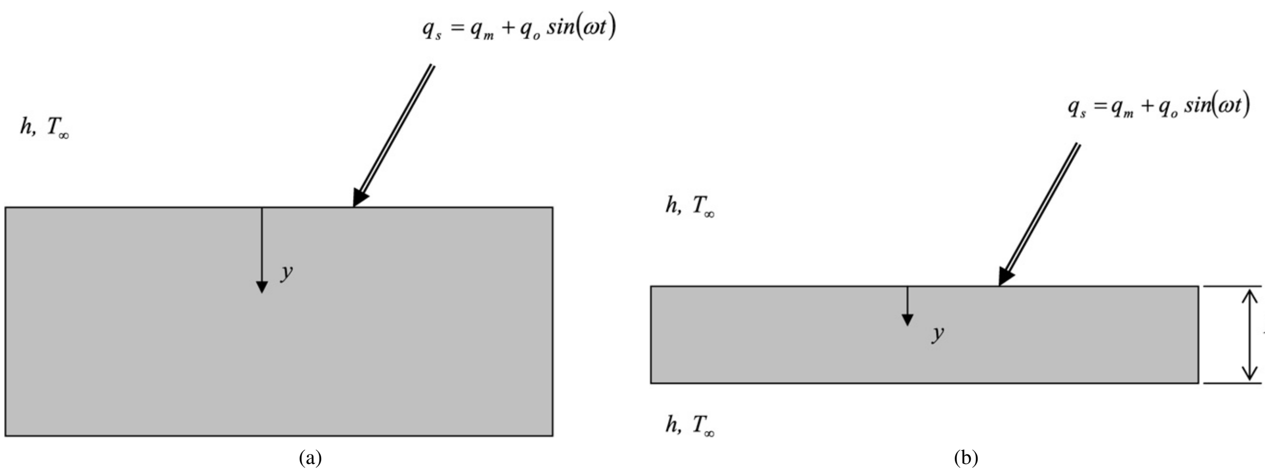

Fig. 1. (a) Schematic diagram for the semi-infinite medium with system coordinate and thermal boundary conditions. (b) Schematic diagram for the finite medium with system coordinate and thermal boundary conditions.

tive boundary condition is obtained for a semi-infinite medium and across a slab wall. Fourier conduction is considered in this work. The Laplace transform method and the principle of superposition are utilized to obtain the mean temperature field and the oscillatory temperature distributions. As such, the rate of heat transferred to the semi-infinite medium and the slab wall is then calculated. Furthermore, analytical solutions for the entropy transfer to both regions are obtained. The controlling parameters are found and their influence on the propagation of the periodic thermal boundary conditions in heat and entropy transfer in both semi-infinite and finite media is explored.

2. Problem formulation

2.1. Conduction through a semi-infinite medium

Consider a semi infinite medium that has an initial temperature of T_0 that identical to the ambient temperature T_∞ ($T_\infty = T_0$). The interface is considered to be subject to convection and periodic heat flux with respect to time boundary conditions as shown in Fig. 1(a). The periodic applied heat flux q_s is taken to have the following idealistic [7] sinusoidal model:

$$q_s = q_m + q_d = q_m + q_0 \sin(\omega t) \tag{1}$$

where q_m, q_d and ω are the applied mean heat flux, the disturbance in the applied heat flux and the frequency of the disturbance in the applied heat flux. The variable t is the time and q_0 is the amplitude of the disturbance in the heat flux. Based on the Fourier's law of conduction, the appropriate heat diffusion equation is

$$\frac{\partial^2\theta(y,t)}{\partial y^2} = \frac{1}{\alpha}\frac{\partial\theta(y,t)}{\partial t} \tag{2}$$

where the excess temperature θ is defined as $\theta = T - T_\infty$. The variable y represents the displacement along the y-axis normal to the surface as shown in Fig. 1(a), $y = 0$ is at the free surface of the medium. The thermal boundary conditions and initial temperature are

$$-h\theta(0,t)+q_s+k\frac{\partial\theta(0,t)}{\partial y}=0;\qquad \left.\frac{\partial\theta}{\partial y}\right|_{y\to\infty}=0$$
$$\theta(x,t=0)=0 \tag{3}$$

where h is the convection coefficient between the ambient and the interface. Since Eq. (2) is a linear partial differential equation with linear coefficients, the solution of θ can be written by the principle of superposition [1,3] as sum of two solutions ($\theta=\theta_m+\theta_d$). The solution θ_m is the mean excess temperature which is due to the influence of the applied mean heat flux q_m. The solution θ_d is oscillatory temperature (or the disturbance temperature) that is due to the effect of the oscillatory part of the applied heat flux.

2.1.1. Solution of the mean excess temperature $\theta_m(y,t)$

The heat diffusion equation, initial temperature and thermal boundary conditions associated with conduction in the semi-infinite medium due to mean part of the applied heat flux are

$$\frac{\partial^2\theta_m}{\partial y^2}=\frac{1}{\alpha}\frac{\partial\theta_m}{\partial t}$$
$$k\frac{\partial\theta_m(0,t)}{\partial y}-h\theta_m(0,t)+q_m=0$$
$$\left.\frac{\partial\theta_m}{\partial y}\right|_{y\to\infty}=0$$
$$\theta_m(x,t=0)=0 \tag{4}$$

The solution θ_m is obtained by the Laplace transform method discussed in Ref. [2,10]. The application of the Laplace transform of θ_m which is defined by

$$\overline{\theta_m}(y,s)=\int_0^\infty\theta_m(y,t)e^{-st}\,dt \tag{5}$$

transforms the diffusion equation and its thermal boundary conditions to

$$\frac{d^2\bar\theta_m}{dy^2}-\frac{s}{\alpha}\bar\theta_m=0$$
$$k\left.\frac{d\bar\theta_m}{dy}\right|_{y=0}-h\bar\theta_m(0,s)+\frac{q_m}{s}=0$$
$$\left.\frac{d\bar\theta_m}{dy}\right|_{y\to\infty}=0 \tag{6}$$

The solution of the transformed equation is then equal to

$$\bar\theta_m(y,s)=\frac{q_m}{h}\left[\frac{e^{-y\sqrt{\frac{s}{\alpha}}}}{s(1+k\sqrt{\frac{s}{\alpha}})}\right] \tag{7}$$

When the inverse Laplace transform [2,10] defined as

$$\bar\theta_m(y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{s\tau}\bar\theta_m(y,s)\,ds \tag{8}$$

(where a is any positive integer) is applied on Eq. (7). The following solution is obtained to the mean excess temperature, mean heat and entropy transferred to the semi-infinite medium, respectively:

$$\theta_m(y,t)=\frac{q_m}{h}\left[1-\frac{1}{\pi}\int_0^\infty\frac{e^{-rt}(\sin(y\sqrt{\frac{r}{\alpha}})+\frac{k}{h}\sqrt{\frac{r}{\alpha}}\cos(y\sqrt{\frac{r}{\alpha}}))}{r(1+(\frac{k}{h})^2\frac{r}{\alpha})}\,dr\right]$$
$$\frac{q_{\text{mean}}}{q_m}=-\frac{k}{q_m}\left.\frac{\partial\theta_m}{\partial y}\right|_{y=0}=\frac{1}{\pi}\int_0^\infty\frac{e^{-(Bi^*)u}}{u^{1/2}(u+1)}\,du \tag{9}$$
$$u=\left(\frac{k}{h}\right)\frac{2r}{\alpha};\qquad Bi^*=\frac{h2t}{\rho ck}$$
$$\dot S_{\text{mean}}=\frac{q_{\text{mean}}}{\theta_m(0,t)+T_\infty}=h\frac{q_{\text{mean}}/q_m}{1/C_1+(1-q_{\text{mean}}/q_m)} \tag{10}$$
$$C_1=\frac{q_m}{hT_\infty} \tag{11}$$

where ρ, c, q_{mean} and $\dot S_{\text{mean}}$ are the density of the medium, specific heat of the medium, heat and entropy transferred to the semi-infinite medium at the interface due the mean value of the applied heat, respectively. Bi^* is the Biot number for the semi-infinite medium and the specific heat of the semi-infinite medium, respectively. The integral in Eqs. (10) and (11) can be evaluated numerically.

2.1.2. Solution for the oscillatory temperature $\theta_d(y,t)$

The heat diffusion equation, initial temperature and the thermal boundary conditions due to the oscillatory portion of the applied heat flux are

$$\frac{\partial^2\theta_d}{\partial y^2}=\frac{1}{\alpha}\frac{\partial\theta_d}{\partial t}$$
$$k\frac{\partial\theta_d(0,t)}{\partial y}-h\theta_d(0,t)+q_0\sin(\omega t)=0$$
$$\left.\frac{\partial\theta_d}{\partial y}\right|_{y\to\infty}=0$$
$$\theta_d(x,t=0)=0 \tag{12}$$

Utilizing the following dimensionless variables

$$\phi=\frac{\theta_d}{\frac{q_0}{k}\sqrt{\frac{\alpha}{\omega}}};\qquad \eta=y\sqrt{\frac{\omega}{\alpha}};\qquad \tau=\omega t \tag{13}$$

The heat diffusion and the boundary conditions reduce to

$$\frac{\partial^2\phi(\eta,\tau)}{\partial\eta^2}=\frac{\partial\phi(\eta,\tau)}{\partial\tau}$$
$$\left.\frac{\partial\phi}{\partial\eta}\right|_{\eta=0,\tau}-\frac{1}{\lambda\sqrt2}\phi(0,\tau)+\sin(\tau)=0$$
$$\left.\frac{\partial\phi}{\partial\eta}\right|_{\eta\to\infty}=0 \tag{14}$$

where λ is referred as to heat transfer parameter. It is equal to

$$\left(\lambda=\frac{k}{h}\sqrt{\frac{\omega}{2\alpha}}\right) \tag{15}$$

The solution of the heat diffusion equation after applying the Laplace transform on it and on its initial and thermal boundary conditions is

$$\bar{\phi}(\eta, s) = \frac{\lambda\sqrt{2}}{(1+\lambda\sqrt{2s})(s^2+1)} e^{-\eta\sqrt{s}} \tag{16}$$

Eq. (16) has simple poles at $s = -i$ and $s = i$ and one branch point at $s = 0$. The steady periodic solution of ϕ is analyzed in this work as it does not vanish with time. It is equal to the sum of the residuals at $s = -i$ and $s = i$ [2,10]. As such, ϕ is equal to the following for large dimensionless times:

$$\phi_{sp}(\eta, \tau) = \lambda\sqrt{2}\left[\frac{(1+\lambda)\sin(\tau - \frac{\eta}{\sqrt{2}}) - \lambda\cos(\tau - \frac{\eta}{\sqrt{2}})}{2\lambda^2 + 2\lambda + 1}\right] e^{-\frac{\eta}{\sqrt{2}}} \tag{17}$$

The dimensionless oscillatory heat flux (heat flux disturbance at the interface) is equal to

$$\Theta(\tau) = \frac{q}{q_0} = -\frac{\partial\phi_{sp}}{\partial\eta}\bigg|_{\eta=0} = \lambda\left[\frac{(1+2\lambda)\sin(\tau) + \cos(\tau)}{2\lambda^2 + 2\lambda + 1}\right] \tag{18}$$

The maximum oscillatory heat flux at the interface occurs at dimensionless times equal to $\tau_{cr1} + 2n\pi$ $(n = 0, 1, 2, \ldots)$ where τ_{cr} is equal to

$$\tau_{cr1} = \tan^{-1}(1+2\lambda) \tag{19}$$

The steady periodic rate at which oscillatory entropy transfer at the interface (disturbance in the entropy transfer) is $\dot{S}_{sp}$ which is equal to (when $q_m = 0$)

$$\Omega = \frac{\dot{S}_{sp}}{k\sqrt{\frac{\omega}{\alpha}}} = \frac{-\partial\phi_{sp}/\partial\eta|_{\eta=0}}{\phi_{sp}(0,\tau) + 1/\Lambda}; \qquad \Lambda = \frac{q_0}{kT_\infty}\sqrt{\frac{\alpha}{\omega}} \tag{20}$$

$$\Omega = \left(\frac{(\lambda + 2\lambda^2)\sin(\tau) + \lambda\cos(\tau)}{[(\lambda+\lambda^2)\sqrt{2}\sin(\tau) - \lambda^2\sqrt{2}\cos(\tau) + \frac{(2\lambda^2+2\lambda+1)}{\Lambda}]}\right)$$

Eq. (20) shows that the rate of oscillatory entropy or the oscillatory heat transferred at the interface is zero when $\tau = \tau_0 + 2n\pi$ $(n = 0, 1, 2, \ldots)$ where τ_0 is equal to

$$\Omega(\tau = \tau_0) = 0 \implies \tan(\tau_0) = -\left(\frac{1}{1+2\lambda}\right) \tag{21}$$

The maximum rate of oscillatory entropy transferred, when $q_m = 0$, at the interface occurs at the dimensionless times $\tau = \tau_{cr2} + 2n\pi$ $(n = 0, 1, 2, \ldots)$ where τ_{cr2} is equal to

$$\frac{d\Omega}{d\tau}\bigg|_{\tau=\tau_{cr2}} = 0 \implies$$
$$\tan(\tau_{cr2}) \cong \frac{(2\lambda+1) - \Lambda\sqrt{2\Lambda(2\lambda+1)^2 + 2 - 4\Lambda^2}}{1 - 2\Lambda^2} \tag{22}$$

2.2. Conduction across a slab wall

Consider a slab wall with uniform thermal properties that has a thickness H and it is initially at a temperature T_0 (identical to the ambient temperature T_∞). One of its surfaces is subjected to convection and a periodic (with respect to time) heat flux as described by model presented in Eq. (1). The convection coefficient is h. The other surface is subjected to convection heat transfer alone with the same convection coefficient h and ambient temperature T_∞. The y-axis is aligned along the wall thickness starting from the heated surface as shown in Fig. 1(b).

2.2.1. Solution to the steady state temperature

The steady state solution for the excess temperature is obtained when $q_d = 0$. As such, the associated heat diffusion equation and boundary conditions are

$$\frac{d^2\theta_m}{dy^2} = 0$$
$$-k\frac{d\theta_m}{dy}\bigg|_{y=0} = q_m - h\theta_m(0)$$
$$-k\frac{d\theta_m}{dy}\bigg|_{y=H} = h\theta_m(H) \tag{23}$$

It can be shown that the mean excess temperature and the mean heat and entropy transferred to the slab wall are the following, respectively:

$$\theta_m(y) = \frac{q_m}{h}\left(\frac{1 + Bi[1 - y/H]}{2 + Bi}\right)$$
$$Bi = \frac{hH}{k} \tag{24}$$

$$\frac{q_{\text{mean}}}{q_m} = \frac{1}{2 + Bi} \tag{25}$$

$$\dot{S}_{\text{mean}} = q_{\text{mean}}\left[\frac{1}{\theta_m(y=0) + T_\infty} - \frac{1}{\theta_m(y=H) + T_\infty}\right] \tag{26}$$

Note that T_∞ is in Kelvin.

2.2.2. Solution to the oscillatory temperature $\theta_d(y, t)$

As discussed before, the governing heat diffusion equation and its initial and thermal boundary conditions due the oscillatory part in the applied heat flux are

$$\frac{\partial^2\theta_d}{\partial y^2} = \frac{1}{\alpha}\frac{\partial\theta_d}{\partial t}$$
$$-k\frac{\partial\theta_d}{\partial y}\bigg|_{y=0,t} = q_0\sin(\omega t) - h\theta_d(0, t)$$
$$-k\frac{\partial\theta_d}{\partial y}\bigg|_{y=H,t} = h\theta_d(H, t)$$
$$\theta_d(y, 0) = 0 \tag{27}$$

The following dimensionless variables are suggested:

$$\tau = \omega t, \qquad Y = \frac{y}{H}; \qquad \phi = \frac{\theta_d}{qH/k} \tag{28}$$

Accordingly, the diffusion equation along with initial and thermal boundary conditions reduce to

$$\frac{\partial\phi}{\partial\tau} = \frac{1}{P}\frac{\partial^2\phi}{\partial Y^2}$$
$$\phi(Y, 0) = 0$$

$$\phi(0,\tau) - \frac{1}{Bi}\frac{\partial\phi(0.\tau)}{\partial Y} = \frac{1}{Bi}\sin(\tau)$$
$$\phi(1,\tau) + \frac{1}{Bi}\frac{\partial\phi(1,\tau)}{\partial Y} = 0 \tag{29}$$

where P and Bi are the thermal oscillation parameter and the Biot number, respectively. They are defined as follow:

$$P = \frac{H^2\omega}{\alpha}, \qquad Bi = \frac{hH}{k} \tag{30}$$

The solution of the heat diffusion equation after applying the Laplace transform [11] is

$$\begin{aligned}\bar{\phi}(Y,s) = &-\left\{\left[\cosh(\sqrt{Ps}) + \left(\frac{1}{Bi}\right)\sqrt{Ps}\sinh(\sqrt{Ps})\right]\right.\\ &\left.\times\sinh(\sqrt{Ps}Y)\right\}\\ &\times\left\{Bi\left(s^2+1\right)\left[\left(1+\left(\frac{1}{Bi}\right)^2 Ps\right)\sinh(\sqrt{Ps})\right.\right.\\ &\left.\left.+2\left(\frac{1}{Bi}\right)\sqrt{Ps}\cosh(\sqrt{Ps})\right]\right\}^{-1}\\ &+\left\{\left[\sinh(\sqrt{Ps}) + \left(\frac{1}{Bi}\right)\sqrt{Ps}\cosh(\sqrt{Ps})\right]\right.\\ &\left.\times\cosh(\sqrt{Ps}Y)\right\}\\ &\times\left\{Bi\left(s^2+1\right)\left[\left(1+\left(\frac{1}{Bi}\right)^2 Ps\right)\sinh(\sqrt{Ps})\right.\right.\\ &\left.\left.+2\left(\frac{1}{Bi}\right)\sqrt{Ps}\cosh(\sqrt{Ps})\right]\right\}^{-1}\end{aligned} \tag{31}$$

Eq. (31) has simple poles at $s = -i$ and $s = i$. These poles are responsible for the steady periodic solution for ϕ. It has infinite number of poles on the negative s-axis that are responsible for the transient decaying solution that will not be discussed here. The steady periodic solution for ϕ_{sp} is calculated using the following equation:

$$\phi_{sp}(Y,\tau) = Res\left[\bar{\theta}(Y,s)e^{s\tau}\right]_{-i} + Res\left[\bar{\theta}(Y,s)e^{s\tau}\right]_{i} \tag{32}$$

where Res designate the residue [10]. The steady periodic temperature is then equal to

$$\begin{aligned}&\phi_{sp}(Y,\tau)\\ &= -\frac{(M_1M_3+M_2M_4)(f_1(Y)\sin(\tau)+f_2(Y)\cos(\tau))}{(M_3^2+M_4^2)Bi}\\ &\quad-\frac{(M_1M_4-M_2M_3)(f_2(Y)\sin(\tau)-f_1(Y)\cos(\tau))}{(M_3^2+M_4^2)Bi}\\ &\quad+\frac{(M_3M_5+M_4M_6)(f_3(Y)\sin(\tau)+f_4(Y)\cos(\tau))}{(M_3^2+M_4^2)Bi}\\ &\quad+\frac{(M_3M_6-M_4M_5)(f_3(Y)\cos(\tau)-f_4(Y)\sin(\tau))}{(M_3^2+M_4^2)Bi}\end{aligned} \tag{33}$$

where the constants M_1, M_2, M_3, M_4, M_5 and M_6 and the functions f_1, f_2, f_3 and f_4 are defined in Appendix A. The dimensionless heat transfer is equal to the following:

$$\Theta = \frac{q}{q_0} = \partial\phi_{sp}/\partial Y|_{Y=H} - \partial\phi_{sp}/\partial Y|_{Y=0} \tag{34}$$

The net rate of dimensionless entropy transferred, when $q_m = 0.0$, to the wall per unit surface area can be calculated from the following:

$$\Omega = \frac{\dot{S}_{sp}}{k/H} = \frac{\partial\phi_{sp}/\partial Y|_{Y=H}}{\phi_{sp}(H,\tau)+1/\Lambda} - \frac{\partial\phi_{sp}/\partial Y|_{Y=0}}{\phi_{sp}(0,\tau)+1/\Lambda}$$
$$\Lambda = \frac{q_0 H}{kT_\infty} \tag{35}$$

3. Discussion of the results

3.1. Discussion of the results of the semi-infinite medium

Fig. 2 shows the effect of the heat transfer parameter λ on the steady periodic dimensionless temperature profile. As can be seen, fluctuations in dimensionless temperature profile near the interface increases as λ increases. The parameter λ increases as the convection coefficient decreases. Decreasing the convection coefficient increases the effect of the applied external heat flux on the semi-infinite medium. That applied heat flux is dissipated to both the medium and the ambient by conduction and convection, respectively. As such, the noise in heat and entropy transfer (amplitude of heat and entropy transferred) to the semi-infinite medium increases as λ increases as shown in Figs. 3 and 4, respectively. Also, these figures show that the time lag between the maximum of heat and entropy transfer and the maximum of the disturbance in the applied heat flux decreases as λ increases which can be noticed from Fig. 6. When λ approaches infinity, the disturbance in the applied heat flux has to be dissipated through the medium which maximizes the transport of the disturbance heat flux through the semi-infinite medium as seen from Fig. 5. Also, this effect results in reducing that time lag to zero which agrees with the results in Fig. 6. The selected range of λ fits many of insulation materials.

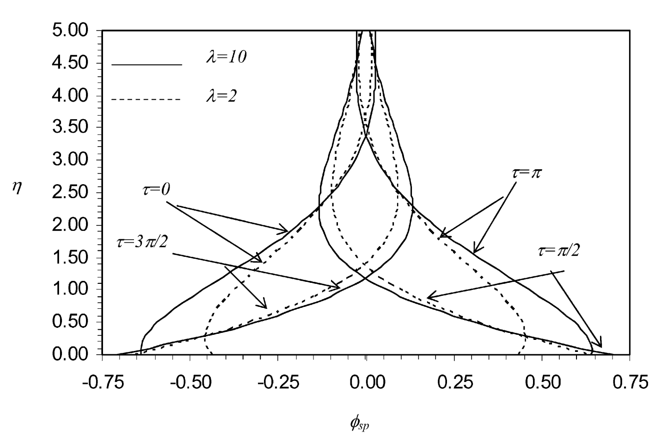

Fig. 2. Effects of the heat transfer parameter λ on the steady periodic dimensionless periodic temperature ϕ_{sp} profile for the semi infinite medium.

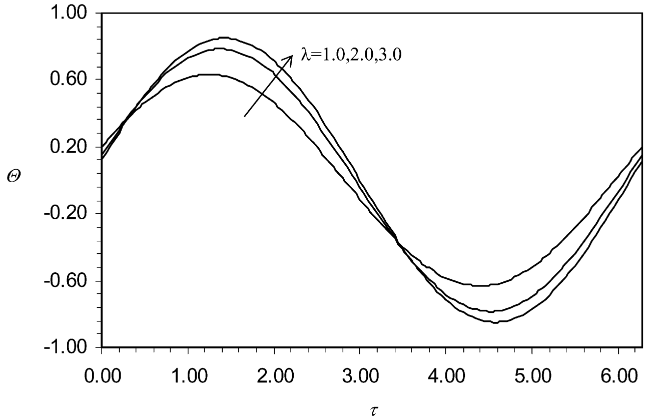

Fig. 3. Effects of the heat transfer parameter λ on the dimensionless oscillatory heat transfer Θ at the interface of the semi infinite medium.

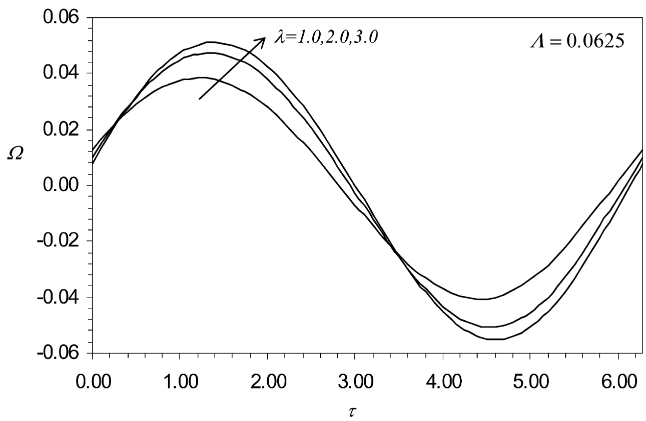

Fig. 4. Effects of the rate of heat transfer parameter λ on the dimensionless rate of entropy transfer Ω at the interface of the semi infinite medium ($q_m = 0.0$).

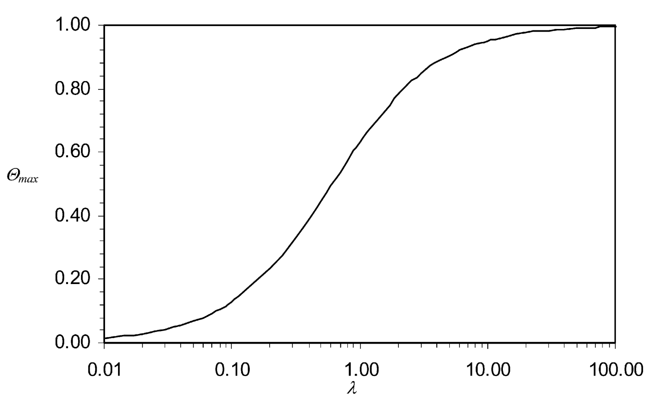

Fig. 5. Variation of the amplitude of dimensionless oscillatory heat transfer $\Theta_{\max}$ with λ for the semi infinite medium.

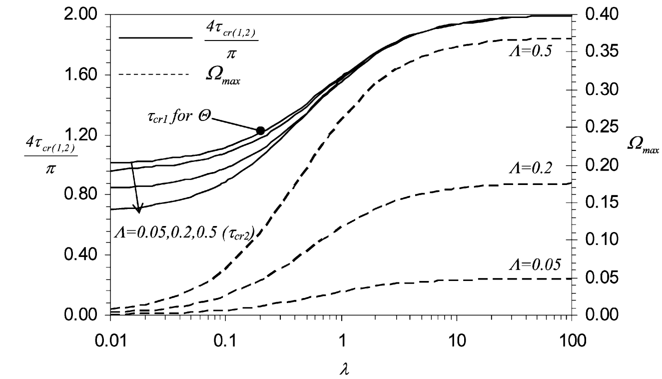

Fig. 6. Variation of the critical times τ_{cr1} and τ_{c2} and $\Omega_{\max}$ with λ and Λ for the semi infinite region ($q_m = 0.0$).

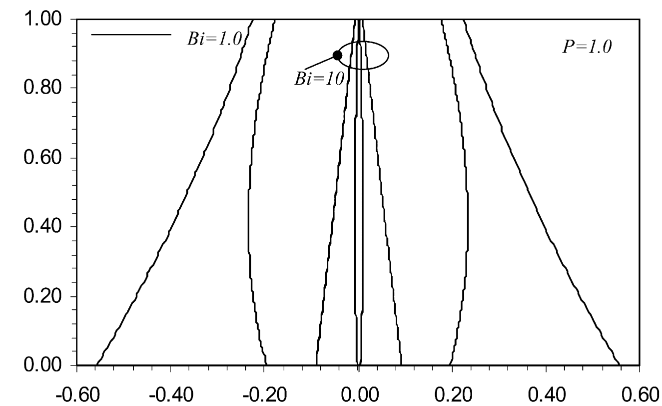

Fig. 7. Effect of the Biot number Bi on the steady periodic dimensionless temperature profile ϕ_{sp} for a slab wall.

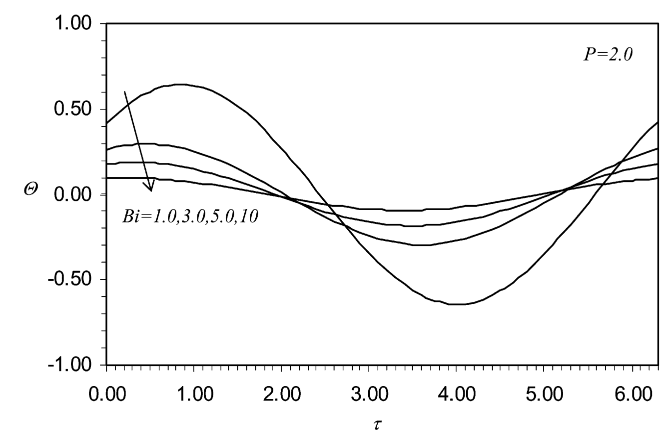

Fig. 8. Effect of the Biot number Bi on the steady periodic dimensionless net heat transferred $\Theta(\tau)$ to the slab wall.

The time that maximizes the rate of entropy transfer is found to be lower than that maximizes heat transfer to the semi-infinite medium as shown in Fig. 6. Increasing the convection coefficient h decreases thermal disturbance in heat transfer inside the semi-infinite medium while it is also, decreases the mean heat transfer to the medium as can be noticed from Eq. (10) and shown in Fig. 12. However decreasing the frequency of the applied oscillatory heat flux ω minimizes thermal disturbances through the medium.

3.2. Discussion of the results of the slab wall

Fig. 7 illustrates that the fluctuation in the dimensionless temperature in the slab wall increases as Bi decreases due to the reduction in rate of heat transferred to the ambient. This results in increasing the noise in the rate of heat and entropy transfer to the slab wall as Bi decreases as shown in Figs. 8 and 9. As the thermal capacitance of the slab wall or the fre-

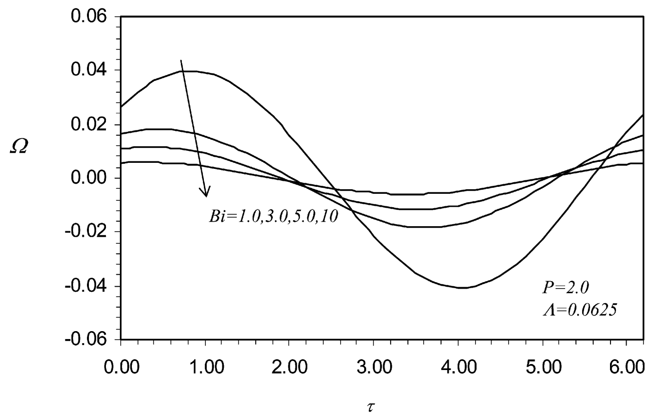

Fig. 9. Effect of the Biot number Bi on the net dimensionless entropy transferred Ω to the slab wall ($q_m = 0.0$).

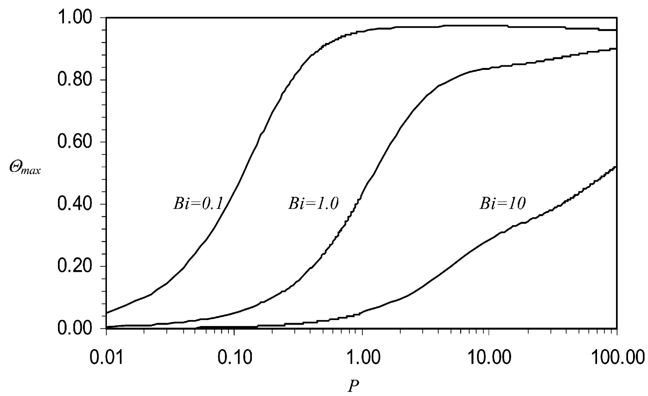

Fig. 10. Effects of the oscillating thermal parameter P and the Biot number Bi on the dimensionless net rate of heat transferred $\Theta_{\max}$ to the slab wall.

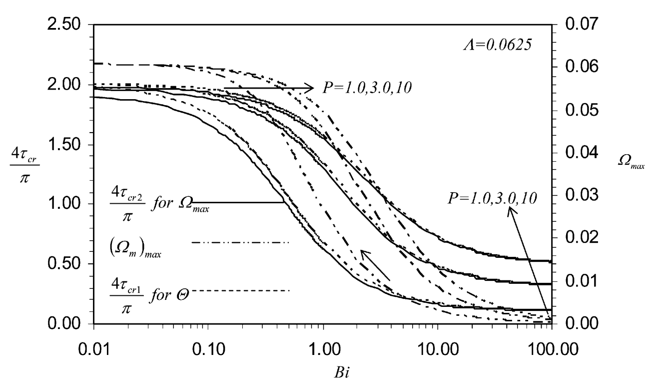

Fig. 11. Effect of the oscillating thermal parameter P on the critical times τ_{cr1} and τ_{c2} and the maximum net rate of entropy transfer $\Omega_{\max}$ to the slab wall ($q_m = 0.0$).

quency of the disturbance increase which can be achieved by increasing the parameter P, the amplitude of the disturbance in heat transferred to the wall increases. Also, the dimensionless times τ_{cr} at which the rate of heat and entropy transferred to the finite medium increases as P increases while it decreases as Bi increases (see Figs. 10 and 11). This indicates that increasing the convection coefficient h and reducing the frequency of disturbance ω minimizes the amplitude of the disturbance in heat and entropy transfer to the wall. However, increasing h decreases the mean heat transfer to the wall as shown in Fig. 12.

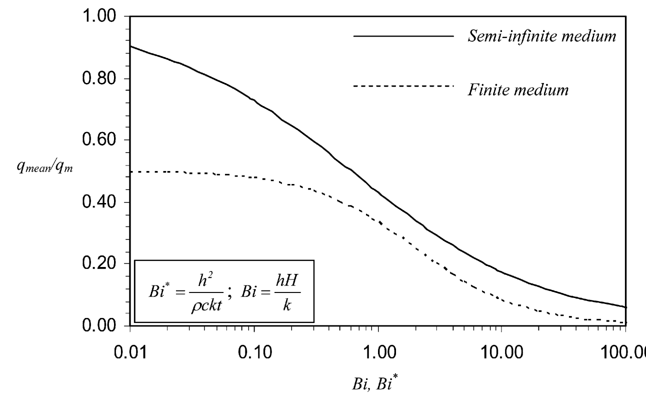

Fig. 12. Effects of Biot numbers Bi and Bi^* on the mean heat transferred to finite and infinite media, respectively.

4. Conclusions

The effects of periodic heat conduction through a semi-infinite medium and a slab wall were studied analytically in this work. The heat diffusion equations for both cases were solved using the Laplace transform method. The Cauchy's residue theorem was used to obtain the inverse Laplace transform of the main components of temperature solution (the mean and oscillatory components). The following concluding remarks can be withdrawn from the discussion section:

- The reduction in k and ω along with an increase in both h and α cause reductions in heat transfer to the semi-infinite medium. This effect diminishes the amplitude of the noise in temperature, heat and entropy transfer due to combined periodic heat flux and convective boundary condition. All previous factors decrease the mean heat and entropy transfer to the semi-infinite medium except the reduction in ω which has no influence on the mean heat and entropy transfer.
- The reduction in k and ω along with an increase in h, H and α cause reductions in heat transfer to the slab wall. This effect diminishes the amplitude of the noise in temperature, heat and entropy transfer due to combined periodic heat flux and convective boundary condition. All previous factors decrease the steady state heat and entropy transfer to the slab wall except the reduction in ω and the increase in α which have no influence on the steady state heat and entropy transfer to the slab wall.

Finally, this work paves a way on controlling the noise in thermal characteristics of solid media and analyzing entropy transfer in periodic conduction applications.

Appendix A

The functions f_1, f_2, f_3 and f_4 are defined as follows

$$f_1(Y)=\sinh\left(\sqrt{\frac{P}{2}}Y\right)\cos\left(\sqrt{\frac{P}{2}}Y\right)$$
$$f_2(Y)=\cosh\left(\sqrt{\frac{P}{2}}Y\right)\sin\left(\sqrt{\frac{P}{2}}Y\right)$$
$$f_3(Y)=\cosh\left(\sqrt{\frac{P}{2}}Y\right)\cos\left(\sqrt{\frac{P}{2}}Y\right)$$
$$f_4(Y)=\sinh\left(\sqrt{\frac{P}{2}}Y\right)\sin\left(\sqrt{\frac{P}{2}}Y\right) \tag{A.1}$$

The constants M_1, M_2, M_3, M_4, M_5 and M_6 are calculated from the following:

$$M_1=\cosh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)+\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)-\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)$$
$$M_2=\sinh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)$$
$$M_3=\sinh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)+2\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)-2\gamma^2\cosh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)-2\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right) \tag{A.2}$$
$$M_4=\cosh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+2\gamma^2\sinh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)+2\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+2\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)$$
$$M_5=\sinh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)+\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right)-\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)$$
$$M_6=\cosh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+\gamma\sinh\left(\sqrt{\frac{P}{2}}\right)\sin\left(\sqrt{\frac{P}{2}}\right)+\gamma\cosh\left(\sqrt{\frac{P}{2}}\right)\cos\left(\sqrt{\frac{P}{2}}\right) \tag{A.3}$$

The parameter γ is equal to

$$\gamma=\left(\frac{1}{Bi}\right)\sqrt{\frac{P}{2}} \tag{A.4}$$

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